Calculation of the Mass Associated to the Photon at Rest

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The wave-energy duality allows us to associate a mass to a photon. Using the energy equation of Einstein's special relativity [11] *associated to a mass and the Plank's energy equation* [12] *associated to a wave, it is possible to associate a mass with a photon of a certain wavelength. In this article, we will show that it is also possible to associate a mass with a photon at rest.*

In 1905, Einstein showed that the moving mass with respect to an observer at rest increases with the Lorentz factor [11]*. Even an infinitely small mass can become important when it approaches the speed of light.*

Over the centuries, physicists (such as Einstein) have realized that there is a speed limit thanks to light. However, nothing implies that light is really the speed limit. In fact, according to our research, this limit seems to be slightly above that of light. The difference between this speed limit and the speed of light is so small that these two speeds seem indissociable.

According to our research, the apparent mass of the universe (about 1.73×10^{53}kg), is due to the kinetic energy of photons that make up our universe. Without their speed, which is that of light in vacuum, the total apparent mass of the universe would in fact be limited to the Planck mass (about 2.1766×10^{-8}kg), which is the mass of a little grain of sand.

KEY WORDS: Mass of the photon, Planck mass, energy, Einstein, speed quantum, speed limit

1. INTRODUCTION

In the standard model, the photon is massless. However, it is possible to associate a mass equivalent to its wave energy thanks to the equations of Einstein [11] and Planck [12]. This mass is small and not measurable.

Although it is entirely <u>hypothetical</u> to conceive a photon at rest, we want to calculate its mass at rest to know the consequences.

2. DEVELOPEMENT

2.1. Value of the Physics Parameters Used

Let's start by stating all the fundamental physics parameters that we intend to use in this article. These values are all available in CODATA 2014 [1].

- Speed of light in vacuum $c \approx 299792458$ m/s
- Universal gravitational constant $G \approx 6.67408(31) \times 10^{-11}$ $m^3/(kg \cdot s^2)$
- Planck length $L_p \approx 1.616229(38) \times 10^{-35}$ m
- Planck mass $m_p \approx 2.176470(51) \times 10^{-8}$ kg

2.2. Postulates on the Maximum Speed of Objects

Thanks to his equations of the special relativity of 1905 [11], Einstein was able to show us that it is impossible for any object to reach the speed of light in the vacuum which is represented by the constant c. If an object of mass m_0 at rest is accelerated at a speed v, it will be perceived as having a mass m' by an observer at rest.

$$m' = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

We find that for a velocity v that tends to c, the mass m' will tend to infinity. At the same time, such a mass would have infinite energy, which is impossible. Therefore, Einstein concluded that it was impossible for a physical object to travel at the speed of light.

In fact, it is impossible to take any object of the universe and accelerate it at such a speed that the mass of the object would have a mass greater than the apparent mass of the universe. In a way, if from the energy point of view, nothing is lost and nothing is created, but everything is transformed, it is easy to accept that we can not give to an object more energy than the energy contained in the universe itself.

Based on these evidences, we postulate the following assumptions:

> **Postulate 1:** It is impossible for a mass m_0 composed of several particles at rest (relative to an observer at rest) to be accelerated at a speed *v* which would give it an apparent mass *m'* (always for the observer at rest) greater than that of the apparent mass of the universe m_u.
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> **Postulate 2:** It is impossible for an elementary particle at rest (relative to a resting observer) to be accelerated at a speed *v* which would give it an apparent mass *m'* (always for the observer at rest) greater than that the Plank mass m_p.

2.3. The Planck Mass is the Highest Energy Level of an Elementary Particle

In order to use this observation later, we want to show that the Planck mass represents the highest energy level for an elementary particle.

For proof, suppose an elementary particle of mass *m*. Its energy can be represented in two ways; by a mass or by a wave.

The energy contained in a mass *m* is given by Einstein's equation [11]:

$$E = m \cdot c^2 \tag{2}$$

The energy contained in the wavelength $\lambda = c/f$ (where *f* is the frequency of the electromagnetic wave) which is associated to it is given by the Planck equation [12]:

$$E = h \cdot f = \frac{h \cdot c}{\lambda} \tag{3}$$

The wave has a radius *r* and the circular surface described by the radius *r* moves linearly and perpendicularly to the surface by describing the shape of a circular spring. As seen from the side, the shape of the spring is seen as a sinusoidal wave. From the front, it is a vector of radius *r* which rotates on the circumference of a circle with an angular velocity equal to $\lambda = 2\pi \cdot r$.

In equation **(3)**, the highest energy level E will be reached by the smallest possible value of λ. This will be the case for a $r = L_p$ where L_p is the Planck length (the smallest unit of length that may exist).

Knowing that $\lambda = 2\pi{\cdot}r$, let us equate equations **(2)** and **(3)**:

$$m \cdot c^2 = \frac{h \cdot c}{2\pi \cdot r} \quad \textbf{(4)}$$

For $r = L_p$:

$$m \cdot c^2 = \frac{h \cdot c}{2\pi \cdot L_p} \quad \textbf{(5)}$$

On a standard manner, the Planck length L_p is given by the following equation:

$$L_p = \sqrt{\frac{h \cdot G}{2\pi \cdot c^3}} \quad \textbf{(6)}$$

Using the equation **(6)** in the equation **(5)** and isolating m, we obtain:

$$m = \sqrt{\frac{h \cdot c}{2\pi \cdot G}} = m_p \quad \textbf{(7)}$$

We find that the mass m obtained in this way is exactly the definition of the Planck mass m_p. We have just shown, as mentioned earlier, that the Planck mass represents the highest level of energy for an elementary particle.

2.4. The Speed Quantum

In previous works, we have already shown that there exists a speed quantum that we have called ε_v. This speed quantum is the smallest unbreakable unit of velocity that exists.

According to a postulate of Einstein's special relativity, the constant c represents the impassable speed limit of light in vacuum. We make a slight modification to the Einstein's postulate when we affirm that even light cannot truly reach c, and that there is a slight difference between the true velocity of light and the speed limit c.

To simplify our language, let us call the true velocity of light v_L to distinguish it from c which is the speed limit. Of course, $v_L \approx c$. For most equations and applications involving special relativity, this assertion is true and justified. But let us look at cases where this is not the case and where it is important to make the difference between v_L and c.

As shown in **(4)** , it is possible to associate a mass to a wave. Let us associate a mass m_{ph} to the lowest energy photon (which has a wavelength equal to the apparent circumference of the universe $\lambda = 2\pi \cdot R_u$):

$$m_{ph} = \frac{h}{2\pi \cdot R_u \cdot c} \approx 2.72 \times 10^{-69} \text{kg} \tag{8}$$

The apparent radius of the luminous universe R_u is given by [5,6,7][1]:

$$R_u = \frac{c}{H_0} \approx 1.28 \times 10^{26} \text{m} \tag{9}$$

H_0 represents the Hubble constant [3]. It can be approximated to about 72.1 km/(s·MParsec) according to the work of Xiaofeng and its team [4].[2]

Let us now take a particle having the same mass as the mass m_{ph} associated to the lowest energy photon. Of course, the photon travels already at a speed extremely close to c. But let us suppose that the chosen particle is at rest and possesses the mass m_{ph} associated to the photon. Let us see with equation **(1)** how fast it is necessary to accelerate this mass to give it the mass of Planck m_p which corresponds to the highest energy level.

$$m_p = \frac{m_{ph}}{\sqrt{1 - \frac{v_L^2}{c^2}}} \tag{10}$$

We see that for a speed of light v_L equal to the speed limit c, we obtain, for an observer at rest, a mass perceived as being equal to the infinite, which is impossible. Of course, on the other hand, we could say that m_{ph} is zero, but again, it would be impossible to get a finite value of m_p. The only way to get away with such an equation is to admit that v_L is slightly less than c by a factor ε_v that we baptise « speed quantum » [8]:

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[1] The name given to the apparent radius of the luminous universe may differ in these articles. Some will call it « Hubble radius», « radius of the universe » and « size of the universe ».

[2] According to our research, the value of $H_0 \approx 72.09548632(46)$ km/(s·MParsec). See the document « *Calculation of the Universal Gravitational Constant G* » on the Internet site www.pragtec.com/physique

$$v_L = c - \varepsilon_v \tag{11}$$

The whole universe is somehow made of photons. These are either in "rectilinear" uniform motion or confined in particles. Indeed, when we perfectly disintegrate matter, we finally obtain only photons. Of course, this presupposes a reorganization of the subatomic particles, but the idea here is that we can associate photons to whatever exist in the universe.

Let us rewrite equation **(10)** using equation **(11)**:

$$m_p = \frac{m_{ph}}{\sqrt{1 - \frac{\left(c - \varepsilon_v\right)^2}{c^2}}} \tag{12}$$

This also gives:

$$m_p = \frac{m_{ph}}{\sqrt{\frac{2\varepsilon_v}{c} - \frac{\varepsilon_v^{\,2}}{c^2}}} \tag{13}$$

Since the value of ε_v is very small, the square of ε_v is negligible:

$$\frac{2\varepsilon_v}{c} >> \frac{\varepsilon_v^{\,2}}{c^2} \tag{14}$$

Therefore, we can rewrite equation **(13)** by making the following approximation:

$$m_p \approx \frac{m_{ph}}{\sqrt{\frac{2\varepsilon_v}{c}}} \tag{15}$$

If we isolate the speed quantum ε_v, we obtain:

$$\varepsilon_v \approx \frac{c}{2} \cdot \frac{m_{ph}^2}{m_p^2} \tag{16}$$

Knowing that the number N is the maximum number of photons of the lowest energy (having a wavelength $\lambda = 2\pi \cdot R_u$) that the universe may contain:

$$N = \frac{m_u}{m_{ph}} \approx 6.30 \times 10^{121} \tag{17}$$

Knowing that the apparent mass of the universe m_u is given by [2]:

$$m_u = \frac{c^3}{G \cdot H_0} \tag{18}$$

It is possible to demonstrate, from the equations **(7)**, **(8)**, **(9)**, **(17)** and **(18)** that:

$$N = \frac{m_p^2}{m_{ph}^2} \tag{19}$$

Therefore, equation **(16)** can be rewritten as follows to give the speed quantum ε_v [8]:

$$\varepsilon_v \approx \frac{c}{2N} \approx 2.34 \times 10^{-114} \text{m/s} \tag{20}$$

We can see that in equation **(11)**, the speed v_L is really very close to c <u>without being identical</u>.

2.5. The Mass m_0 of the Photon Owing the Lowest Energy at Rest

Let us try to determine the mass m_0 which can be associated to a photon at rest. This is, of course, a hypothetical concept, because nobody has managed to put a photon at rest. To perceive such a mass, an observer should be moving in parallel and at the speed of the photon, which represents another impossibility...

Suppose now that we associate a mass m_{ph} to a moving photon and that the mass associated to the photon at rest is m_0.

$$m_{ph} = \frac{m_0}{\sqrt{1 - \frac{v_L^2}{c^2}}} \tag{21}$$

Let v_L be replaced by equation **(11)** and let's isolate the value of the mass of the photon at rest m_0:

$$m_0 = m_{ph} \cdot \sqrt{1 - \frac{(c - \varepsilon_v)^2}{c^2}} \quad \textbf{(22)}$$

By a process of approximation similar to what was done to obtain the speed quantum ε_v, we obtain:

$$m_0 \approx m_{ph} \cdot \sqrt{\frac{2\varepsilon_v}{c}} \approx \frac{m_{ph}}{\sqrt{N}} \approx 3.45 \times 10^{-130}\,\mathrm{kg} \quad \textbf{(23)}$$

It is a mass so small that it is totally impossible to measure it.

What is even more interesting is that if we multiply it by the maximum number of photons of the lowest energy in the universe, we obtain exactly the Planck mass m_p:

$$m_0 \cdot N \approx \frac{m_{ph}}{\sqrt{N}} \cdot N = m_{ph} \cdot \sqrt{N} = m_p \quad \textbf{(24)}$$

Therefore, if no kinetic energy had been injected in any way into the universe of departure at the big bang, the universe would have had the Planck mass m_p. This is about the mass of a little grain of sand. It is the relativistic effects due to the incredible speed of the photons that makes the universe having now the impressive mass that we know it.

Equation **(24)** is especially interesting when we know that the Planck mass m_p is the geometric mean between the apparent mass of the universe m_u and the mass of the lowest energy photon m_{ph} (having a wavelength $\lambda = 2\pi \cdot R_u$). This equation can be demonstrated from equations **(7)**, **(8)**, **(9)** and **(17)**:

$$m_p = \sqrt{m_u \cdot m_{ph}} \quad \textbf{(25)}$$

This means that the Planck mass m_p plays different roles:

- It is the mass of a particle that has the highest level of energy.
- It is the mass corresponding to the geometric mean between the mass associated to a photon and the apparent mass of the universe.
- It is the total mass of all photons at rest in the universe.

3. CONCLUSION

The interest of this article is to show that it is possible to associate a mass with a photon at rest. We can then observe that the apparent mass of the universe is a relativistic effect due to the speed of the photons.

The quantum velocity ε_v makes it possible to limit the "increase" of the mass of a particle, by relativistic effects due to the velocity, to a mass equal to the Planck mass.

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